Periodische Graphen und ein Resultat von Tutte

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Olaf Delgado-Friedrichs

Wilhelm-Schickard-Institut für Informatik, Universität Tübingen

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(Test in $O(d^2 \cdot |E|)$ [Cohen, Megiddo 1990])

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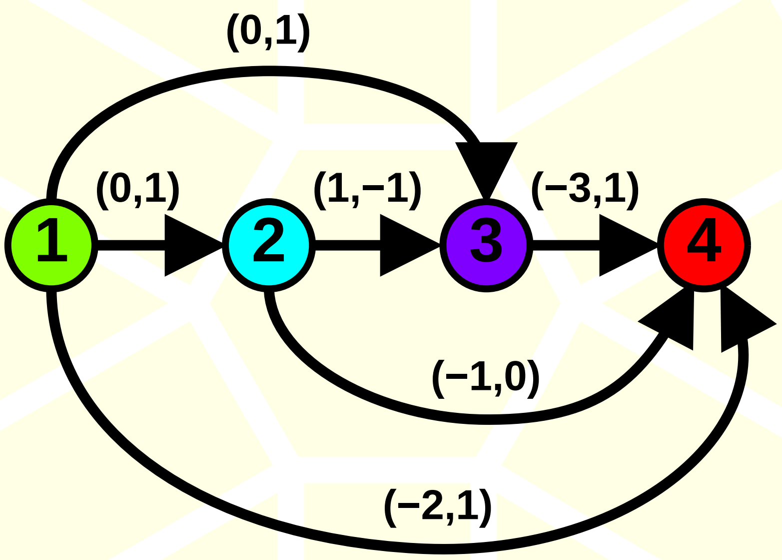
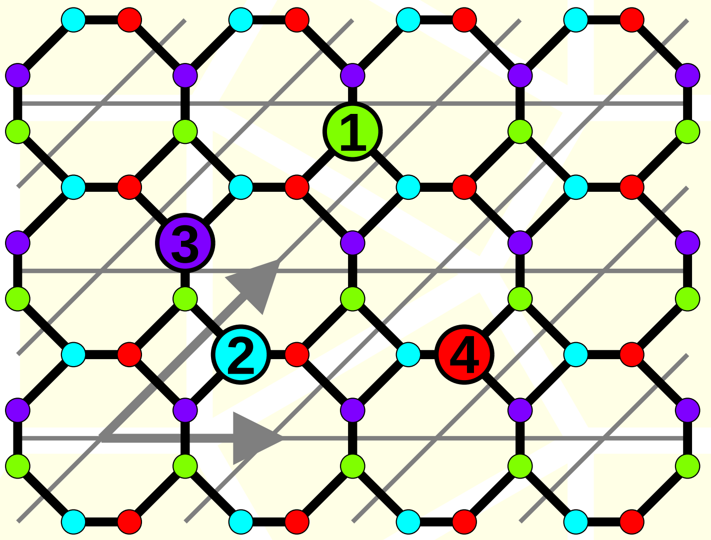
Example 1:



G (the orbit graph)



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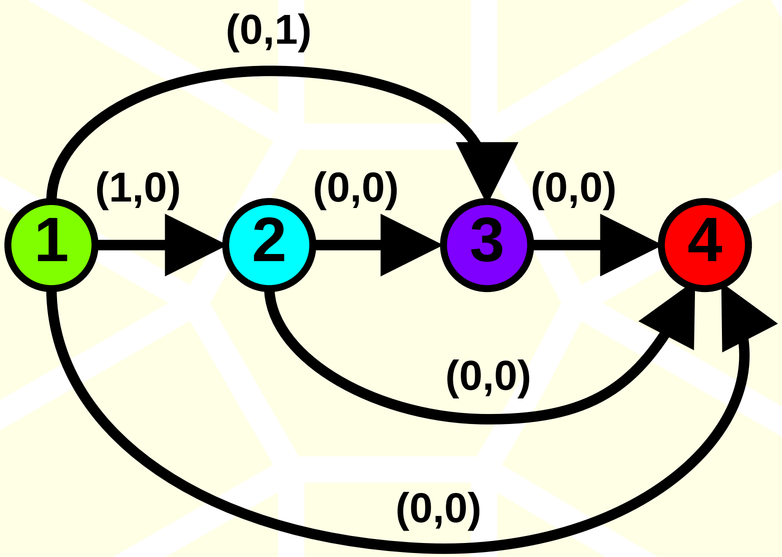

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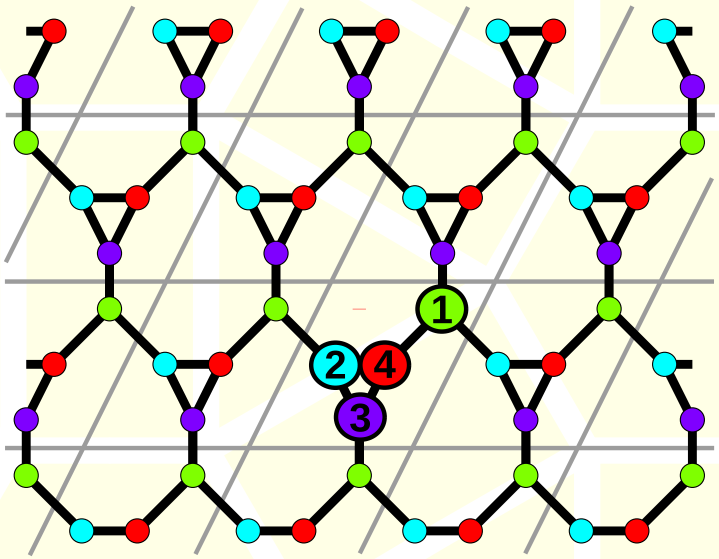
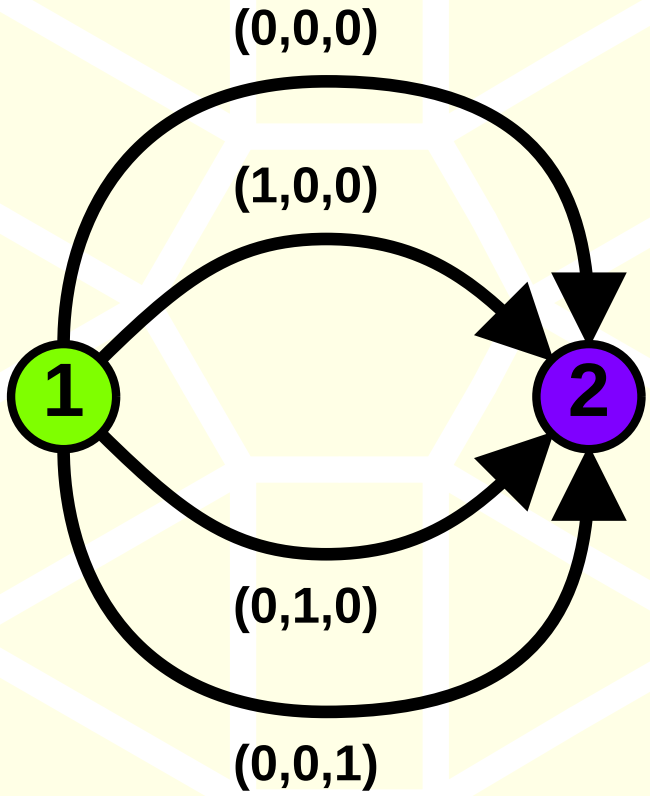


The p-graph.

Example 3: diamond



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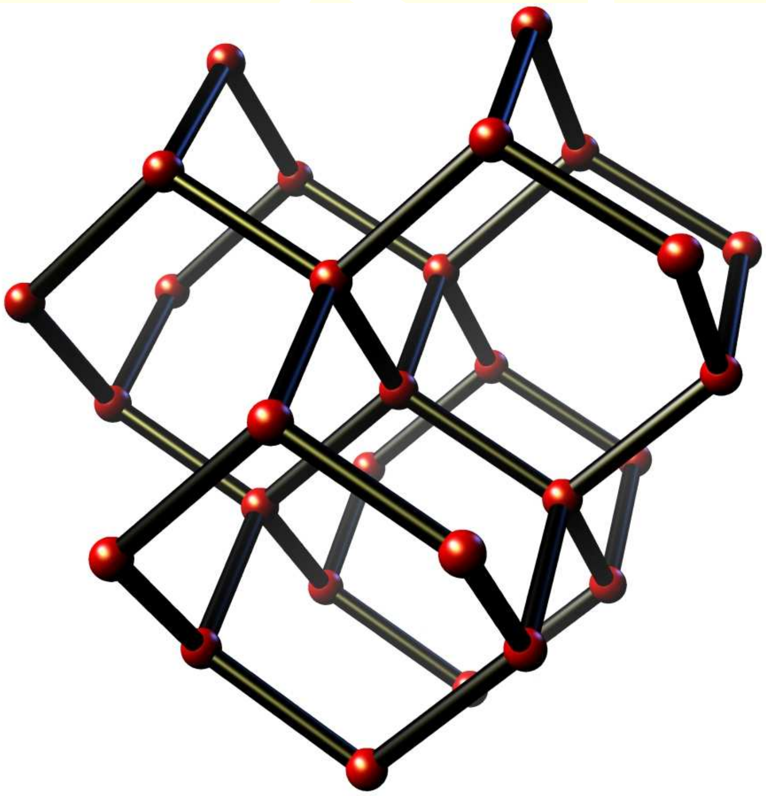

The p-graph.

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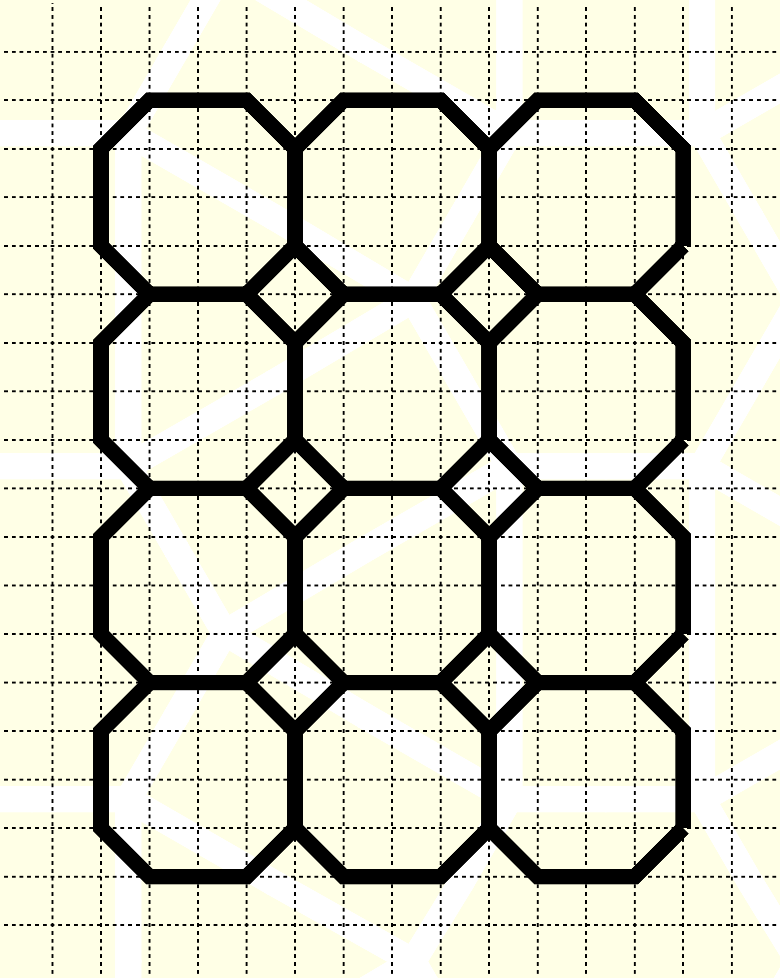
- Periodic drawings defined similarly.

Barycentric drawings

Place each vertex in the center of gravity of its neighbors:



Barycentric drawings

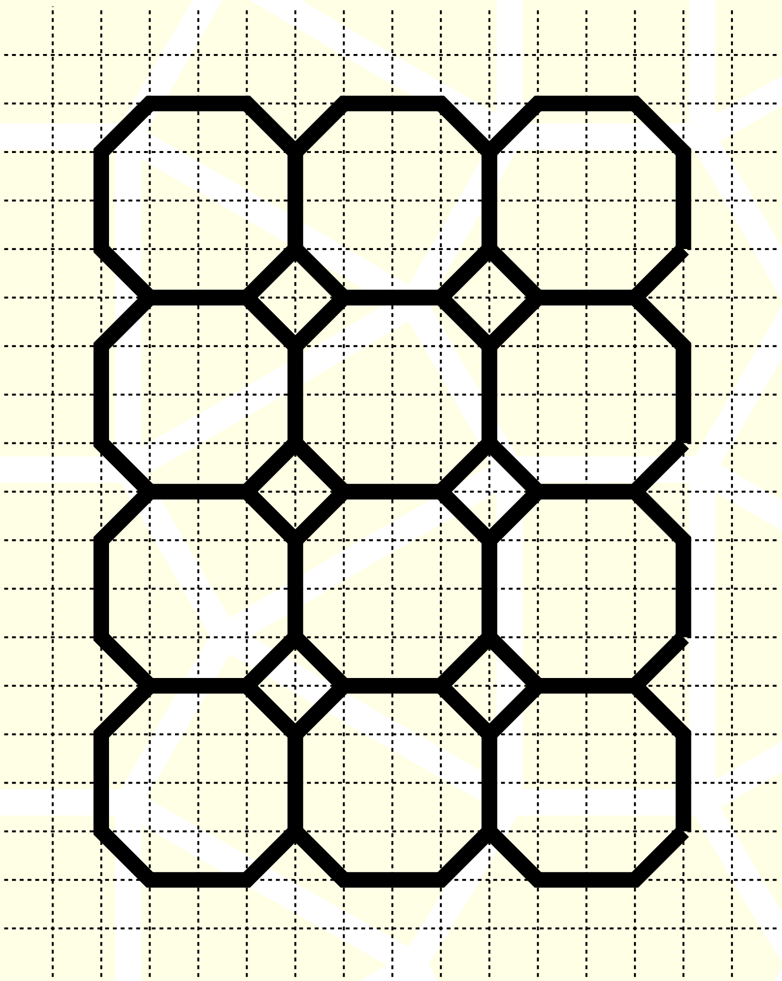
Place each vertex in the center of gravity of its neighbors:

$$p(v) = \frac{1}{d(v)} \sum_{vw \in E^*} p(w)$$

where

$p =$ placement,

$d =$ degree.



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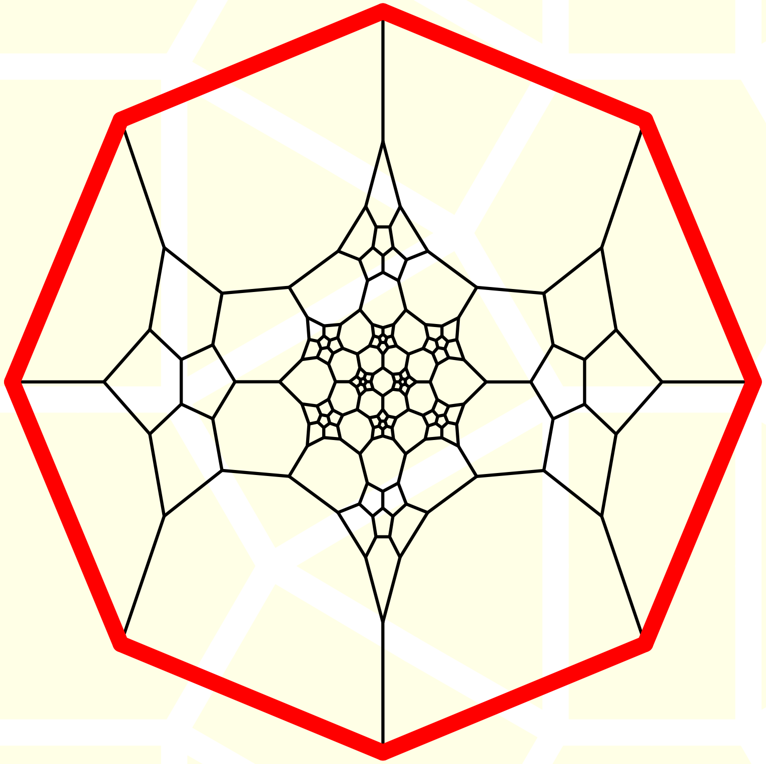
Barycentric drawings

[TUTTE 1960/63]:

- Pick and realize a convex outer face.
- Place rest barycentrically.



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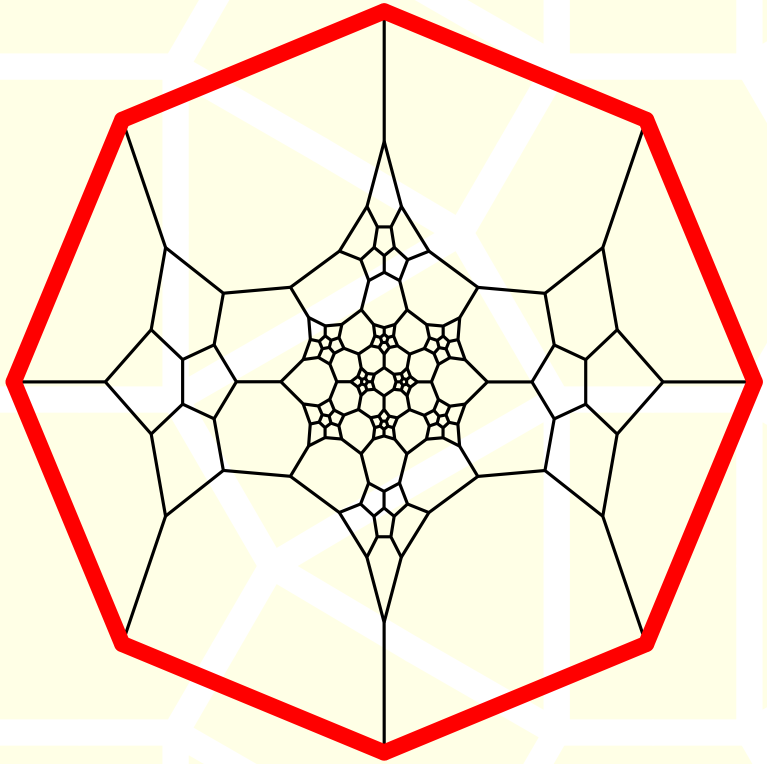

G planar, 3-connected $\Rightarrow$ convex planar drawing.

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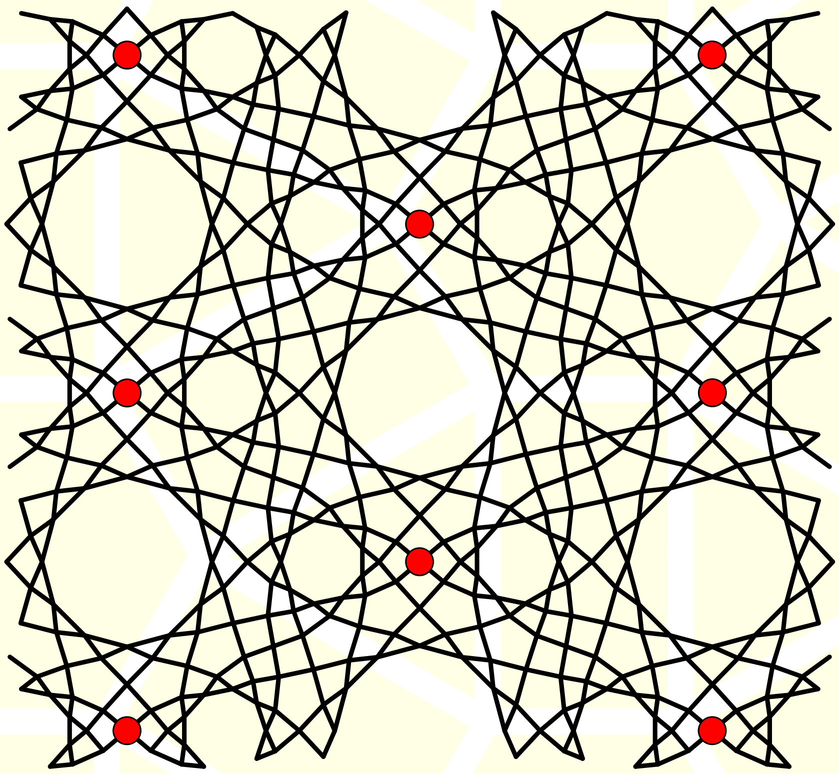
Periodic version

Place one vertex, choose linear map $\mathbb{Z}^d \to \mathbb{R}^d$.



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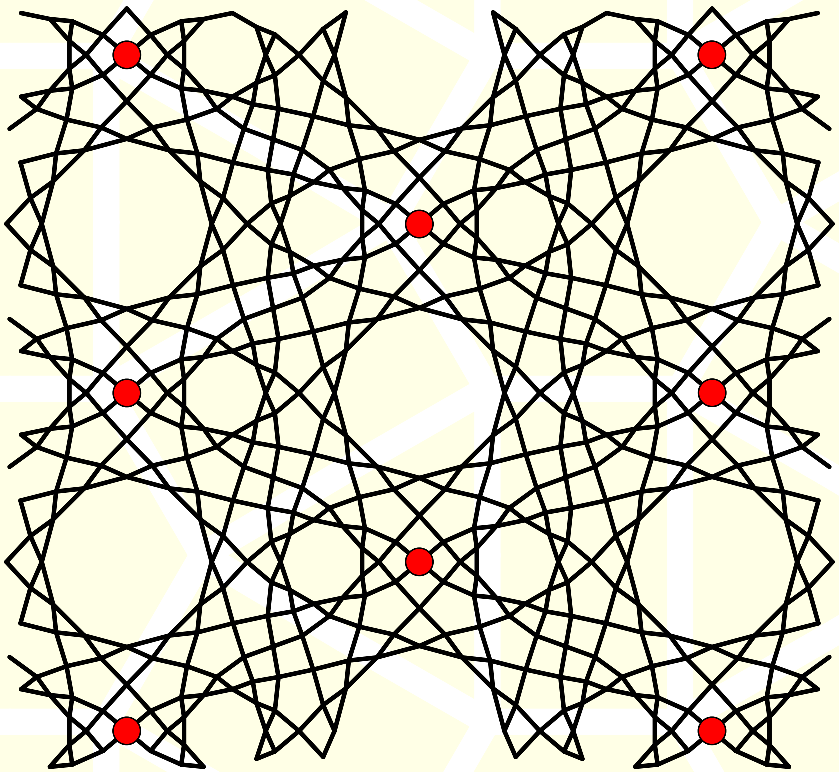

Theorem:
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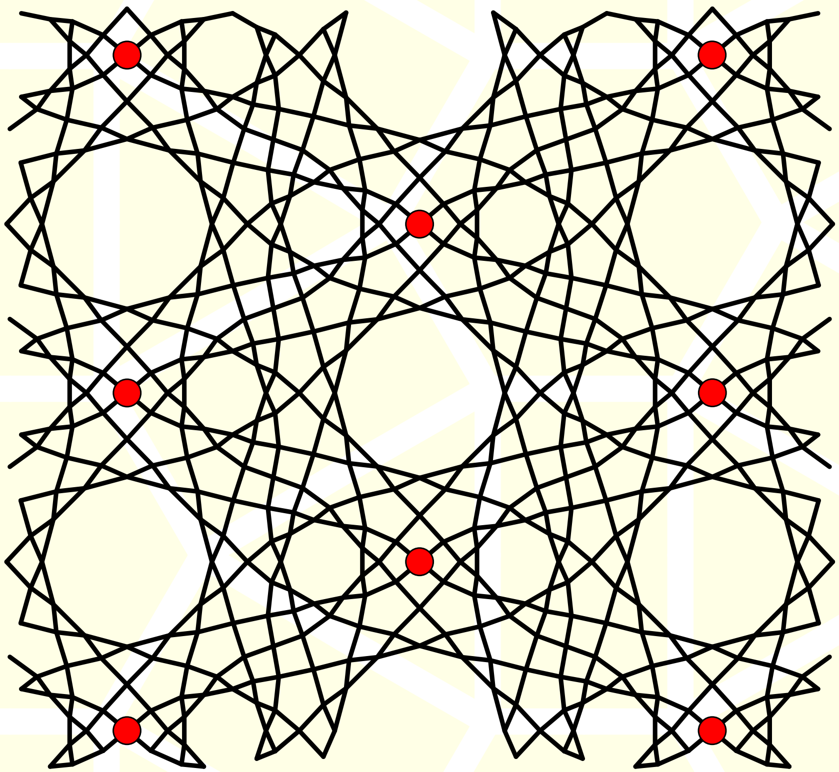

Theorem:
This defines a unique barycentric placement.

Corollary:
All barycentric placements of a p-graph are affinely equivalent.

Sketch of proof
(c.f. [Richter-Gebert 1996])

- Barycentric placements are critical points of

$$W(p) := \sum_{vw \in E} ||p(w, \mathbf{s}(vw)) - p(v, \mathbf{0})||^2.$$

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Remark: Minimizes square-sum of edge lengths.

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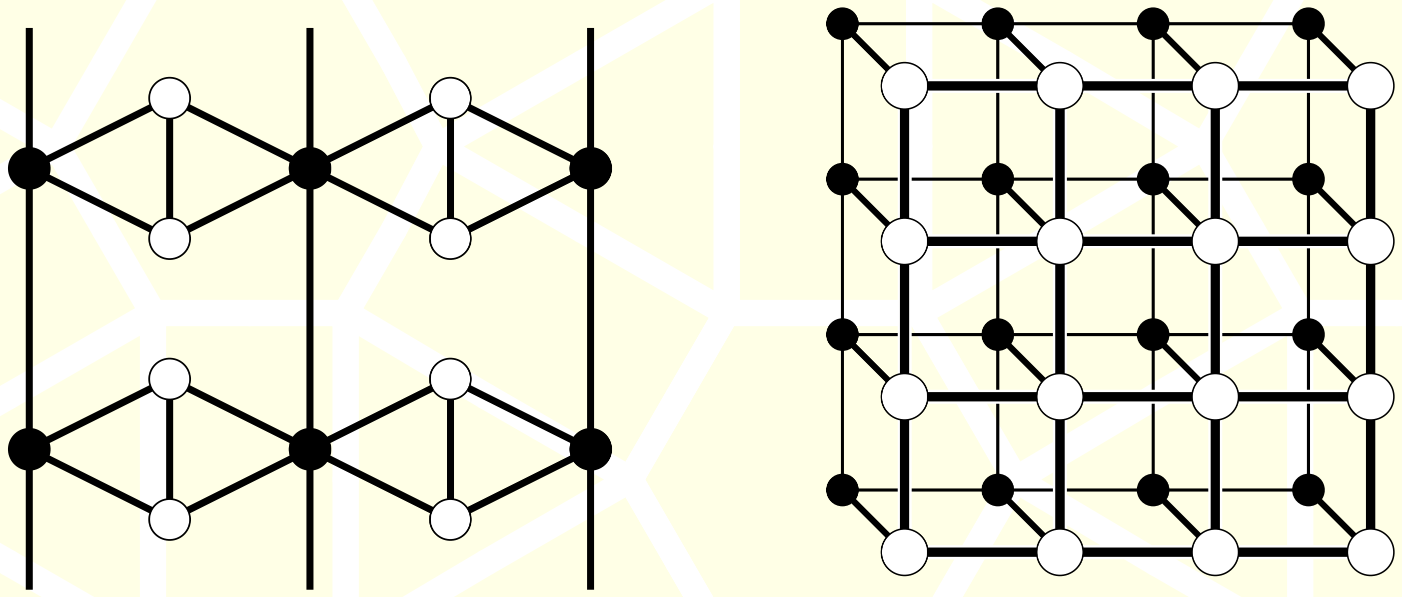
Caveat

The barycentric placement need not be injective:



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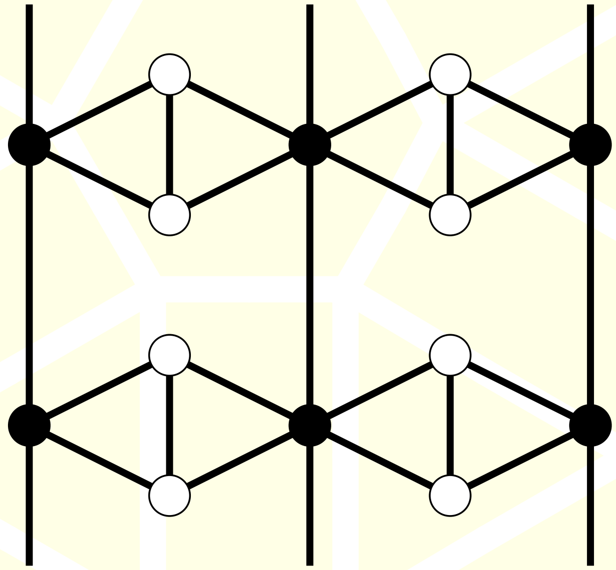

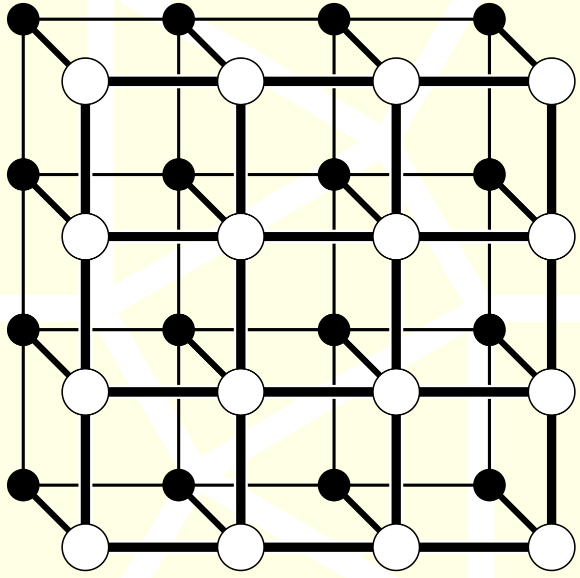

If it is, the p-graph is called stable.

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- Uniqueness $\Rightarrow$
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- ∃ a metric turning all these into isometries.
- ⇒ The automorphism group maps
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- For stable graphs the map is an isomorphism.

⇒ Stable p-graphs can be drawn fully symmetric.

Convex drawings

- [THOMASSEN 1980]: Any “sufficiently nice” infinite, planar, 3-connected graph has a topologically unique planar embedding, which can be made convex. (c.f. [TUTTE 1960/63]).

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- **Corollary:** These graphs are necessarily stable, so they have a convex planar embedding with full symmetry.

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(following [RICHTER-GEBERT 1996])

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(6) Placement induces embedding. (by (4),(5))

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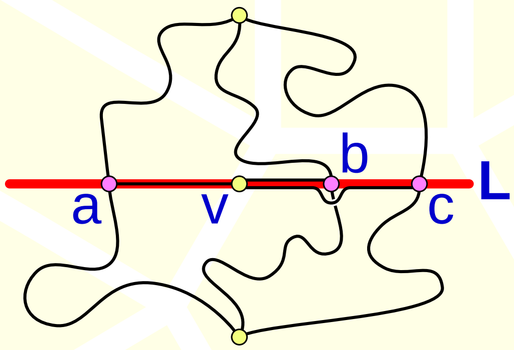
Proof of (2)

(non-degenerate neighborhoods)

Claim: If all neighbors of some vertex v are on a straight line L, then the graph contains a $K_{3,3}$.



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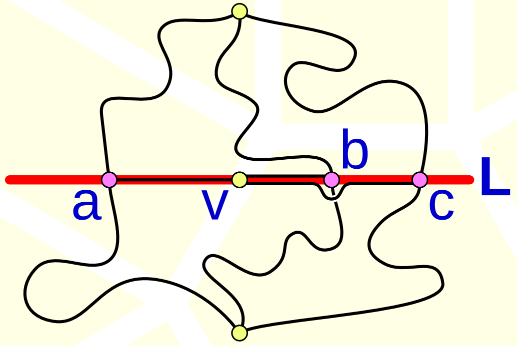

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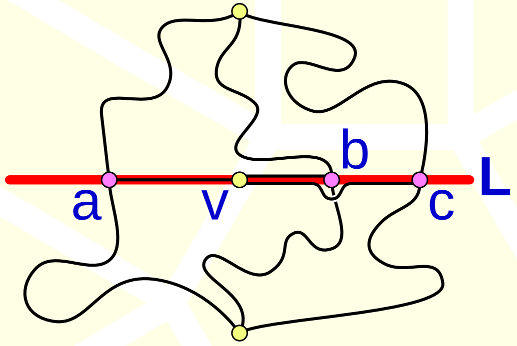

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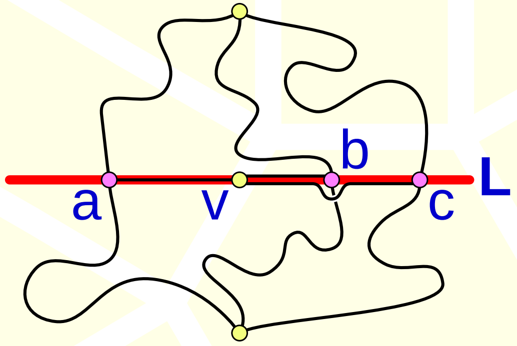

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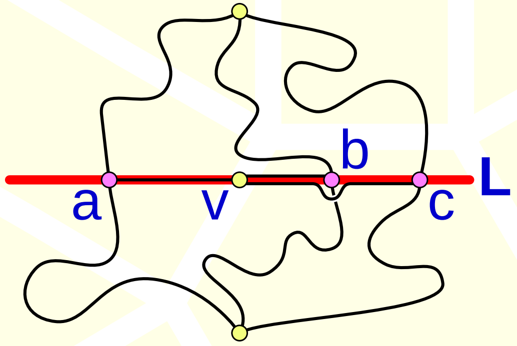

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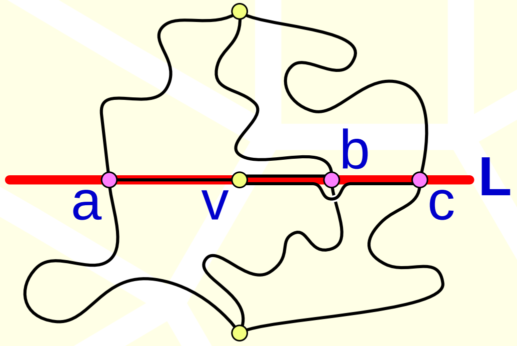

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(e) Repeat construction below L.

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(connection above L)

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Ascending path must reach outer polygon.

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Alternative: construct an artificial "outer polygon".

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- φ is covering $\Rightarrow$ lift to $\mathbb{R}^2$ is homeomorphism.

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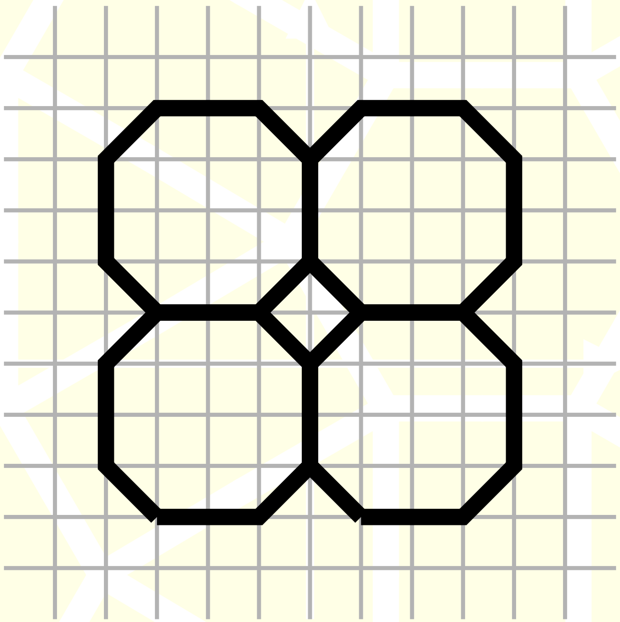
- Idea: generate a small characteristic collection of representations and pick the lexicographically smallest.

Ordered traversals

For G^* locally stable:



Ordered traversals

For G^* locally stable:

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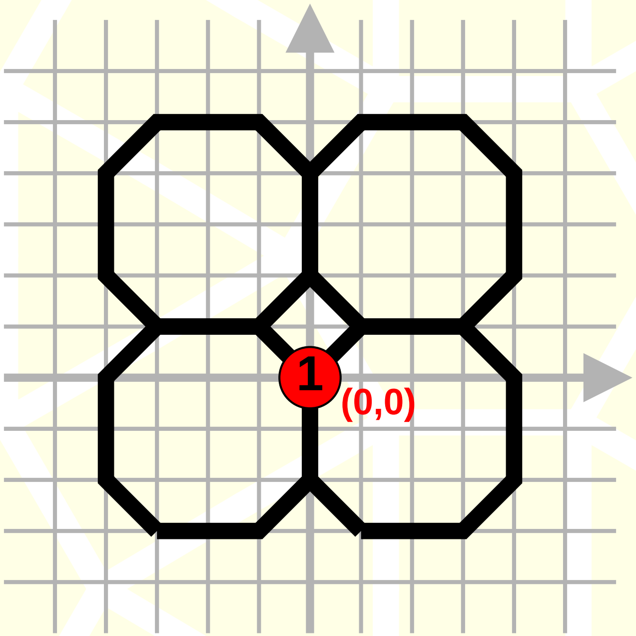

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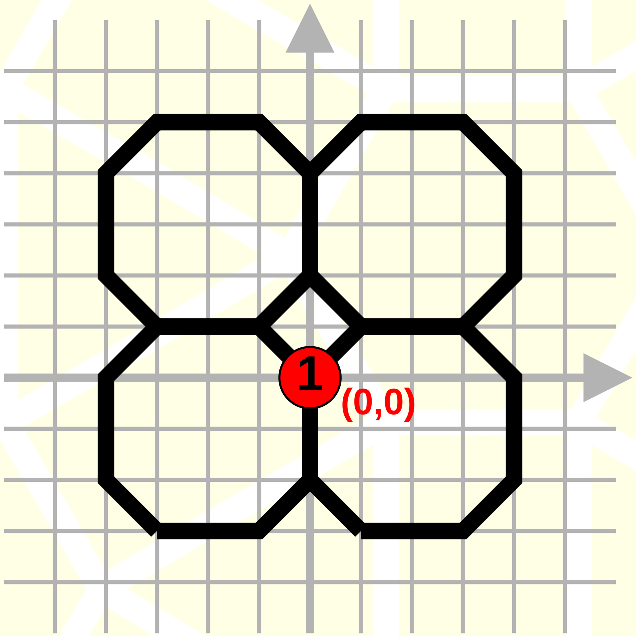

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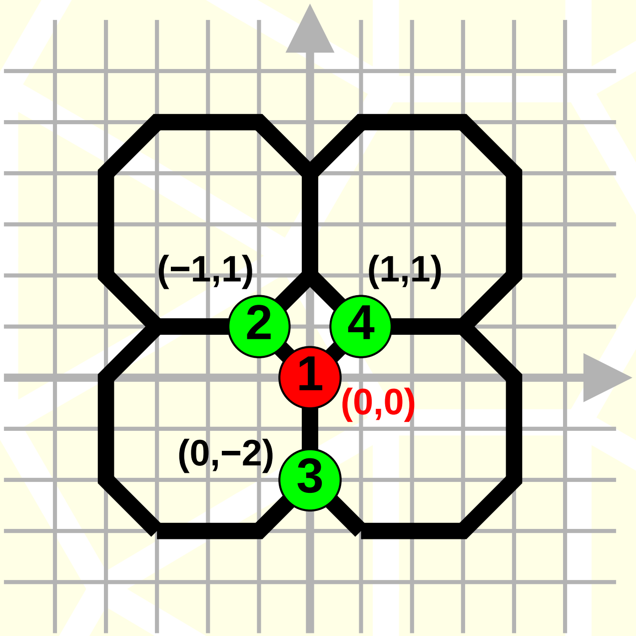

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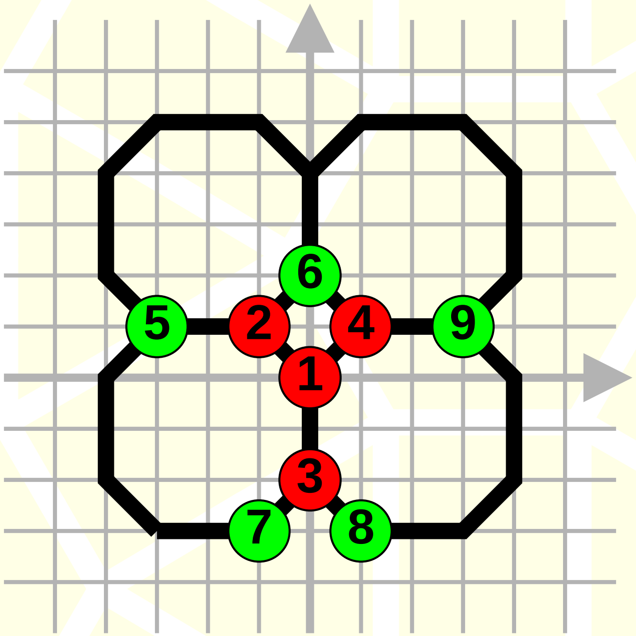

$\Rightarrow$ unique vertex numbering for each set of start conditions.

The traversal construction

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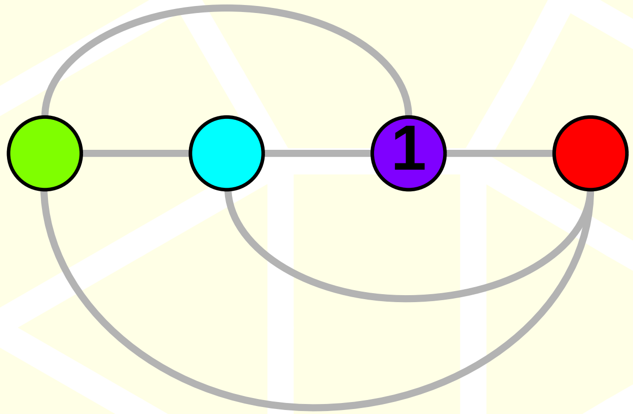

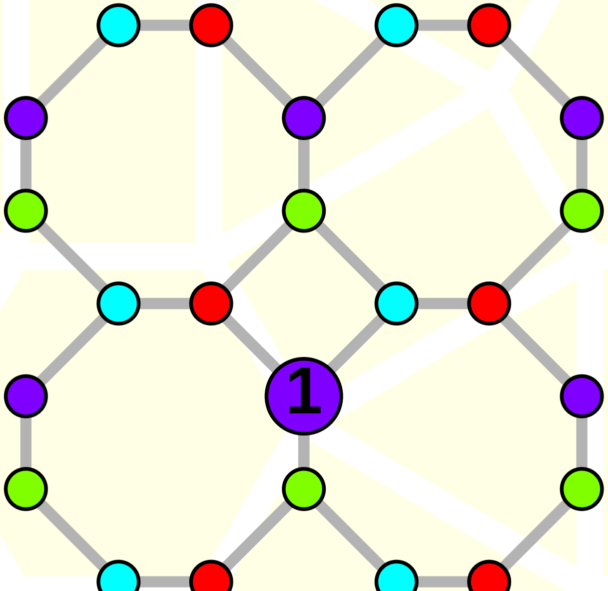

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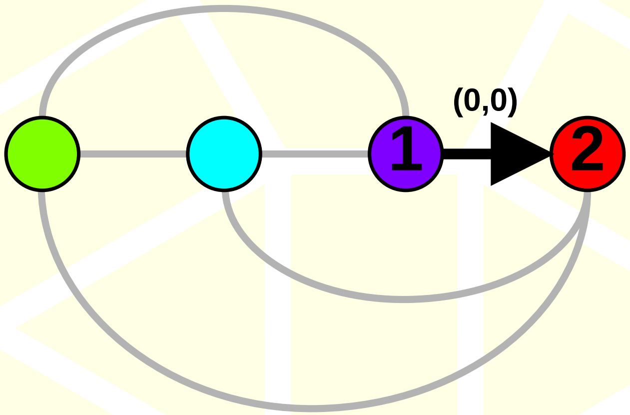

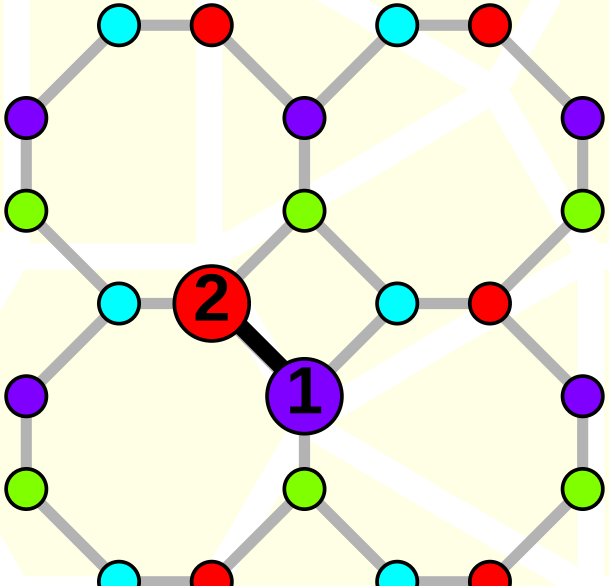

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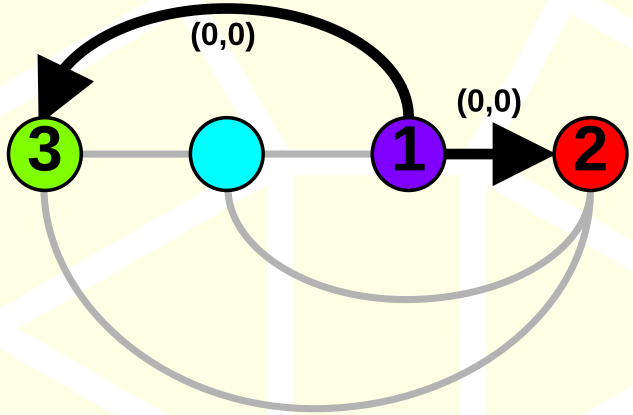

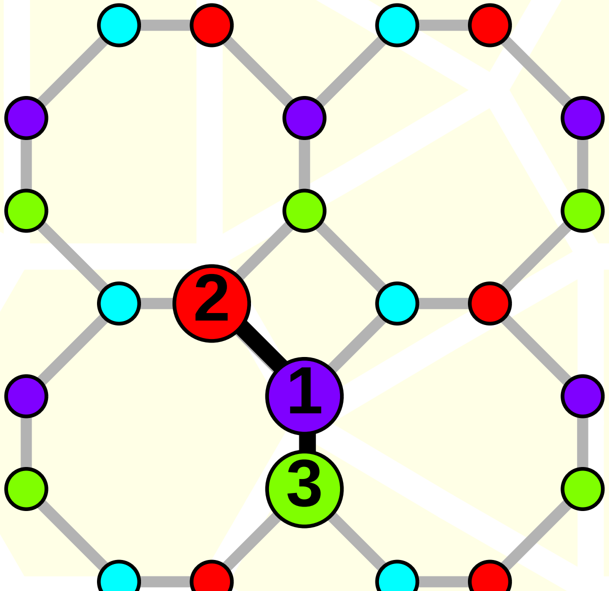

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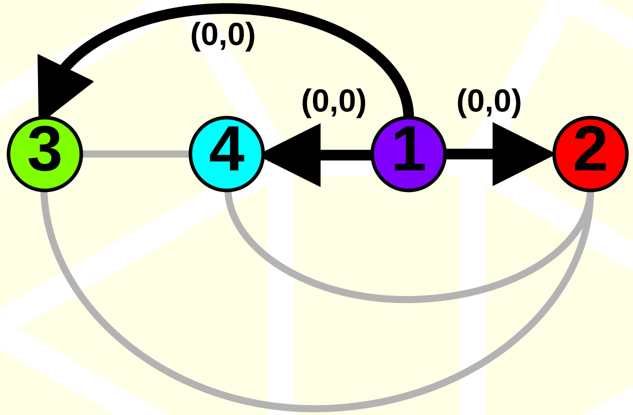

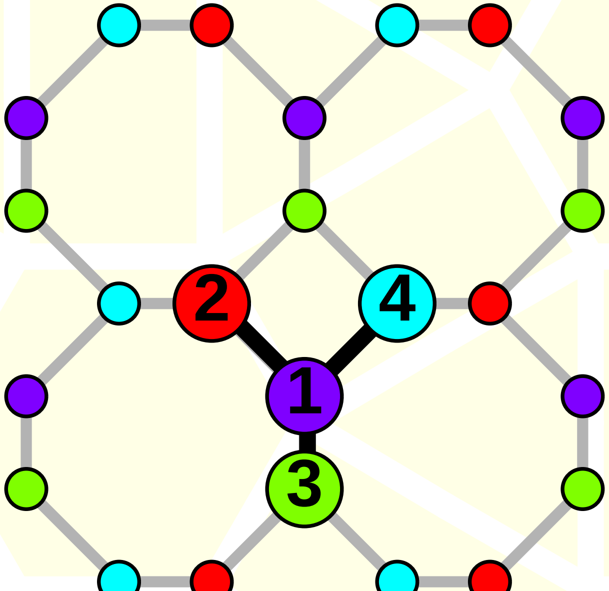

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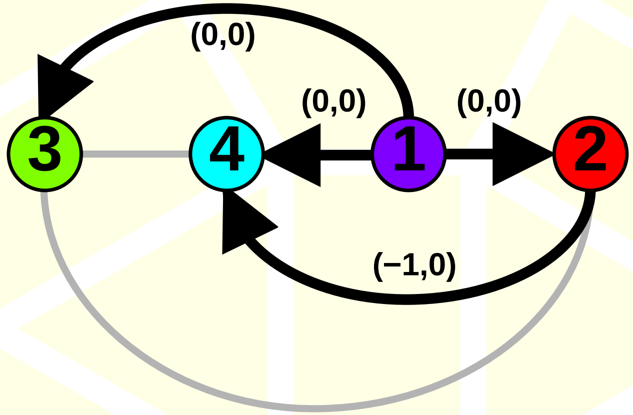

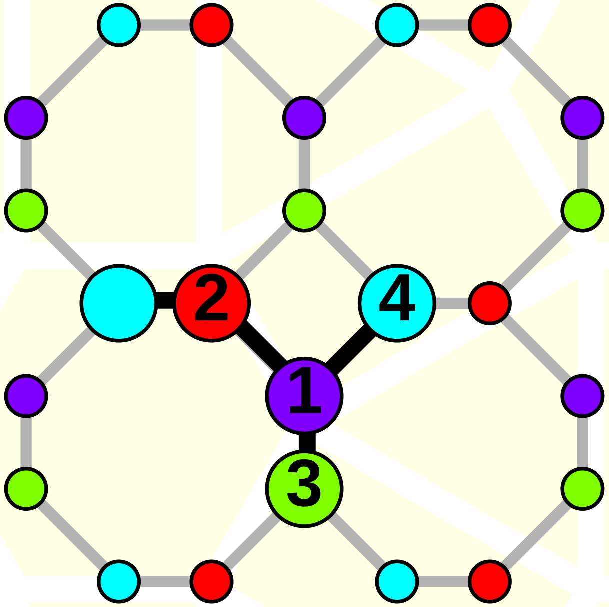

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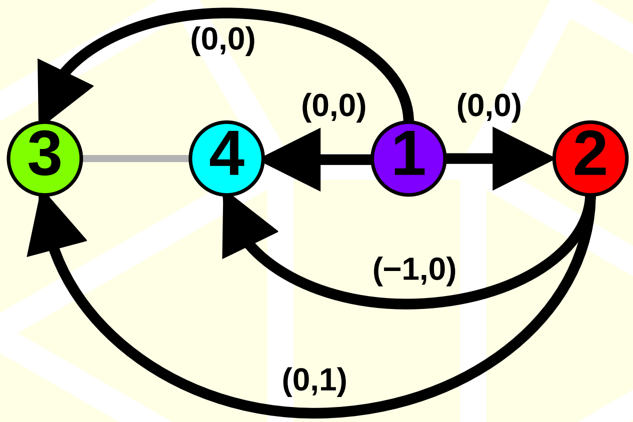

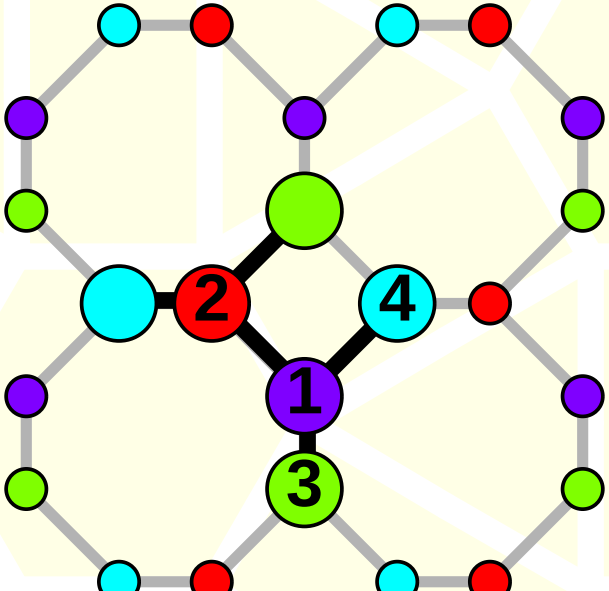

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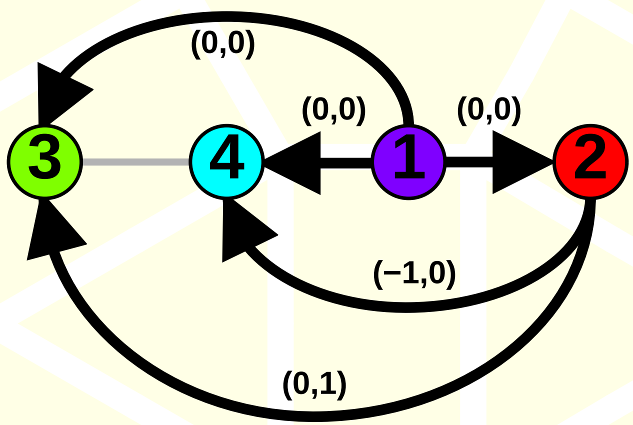

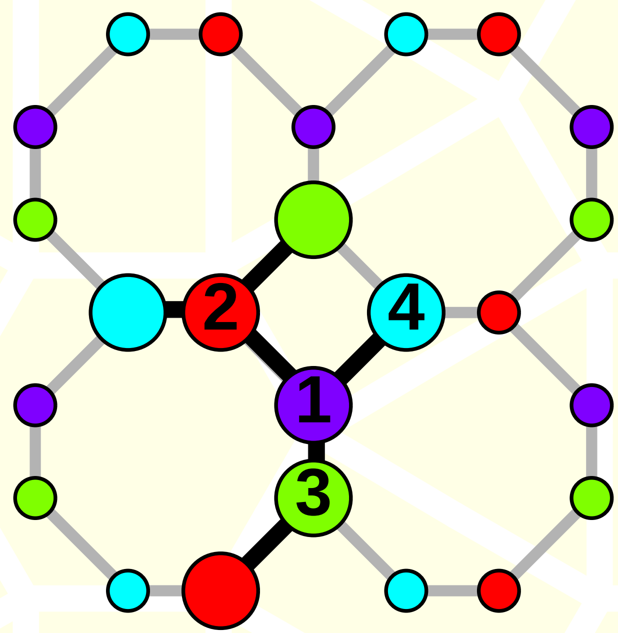

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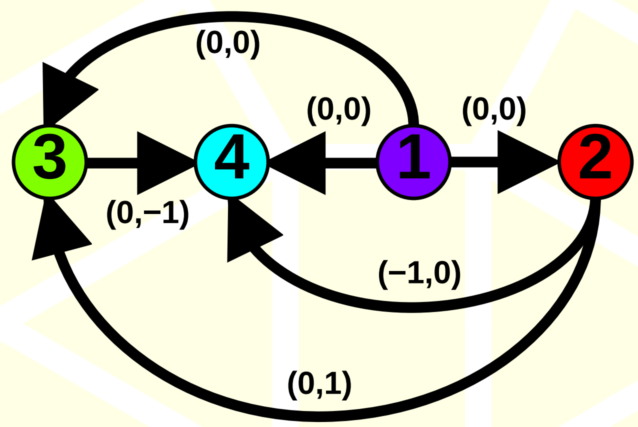

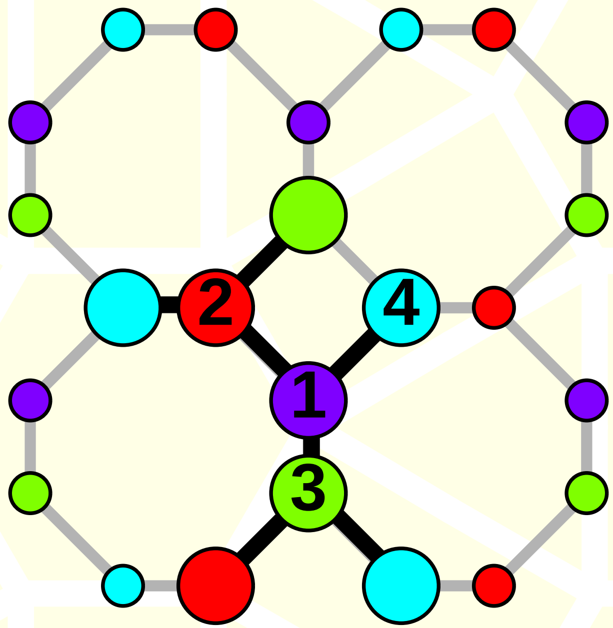

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Thanks for your attention!